A Note on Two Modal Propositions of Burleigh

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In *De Puritate Artis Logicae Tractatus Brevior*, Burleigh affirms the following propositions:

(1) Impossibile potest esse verum.
("The impossible can be true")[1].

(2) Quod potest esse possibile est impossibile.
("That which can be possible is impossible").

I contend that these propositions, subject to my understanding of Burleigh's interpretation of them, entail, respectively, the denial of the characteristic axiom schema of S5 (i.e., $M\alpha \supset LM\alpha$), and of the characteristic axiom schema of S4 (i.e., $L\alpha \supset LL\alpha$).

1. Denial of the characteristic axiom schema of S5

In explicating the sophism associated with (1), Burleigh notes in the *solutio* that:

Prima est multiplex secundum compositionem et divisionem. In sensu compositionis est falsa, et denotatur, quod haec est possibilis: 'Impossibile est verum'. In sensu divisionis est ulterius multiplex, ex eo quod impossibile potest accipi pro eo, quod est impossibile, et sic est falsa, vel pro eo, quod potest esse impossibile, et sic est vera[2].

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[1] All translations are mine.

[2] W. Burleigh, *De Puritate Artis Logicae Tractatus Longior, with a revised edition of the Tractatus Brevior*, Franciscan Institute, St. Bonaventure, New York 1955, pp. 240-1.

The first [proposition] is ambiguous according to composition and division. In the sense of composition it is false and denotes that this is possible: "The impossible is true". In the sense of division it is again ambiguous since that which is impossible can be accepted for that which is impossible, and thus it is false, or [it can be accepted] for that which can be impossible, and thus it is true.

If I understand this claim correctly, a more perspicuous version of (1) would be:

(1')   Quod potest esse impossibile potest esse verum.
       ("That which can be impossible can be true").

What logical sense are we to make of (1')? It seems that Burleigh wants to say that some proposition can, at the same time, be both possibly impossible and possibly true – that is, merely possible. Formulated in a formally stated object language, (1') becomes:

(1'')   M~Mα • Mα

where 'M' denotes the possibility operator and 'α' denotes some proposition.

If (1'') is a correct formalization of (1'), it is easy to verify that this entails the denial of the characteristic axiom schema of S5 as follows:

{1}	1. M~Mα • Mα	assumption
{1}	2. ~LMα • Mα	def. of 'M' and double negation
{1}	3. Mα • ~LMα	commutation
{1}	4. ~(Mα ⊃ LMα)	negation of conditional
		Q. E. D.

2. Denial of the characteristic axiom schema of S4

Typically, in a mediaeval sophism, one adduces a *probatio* (proof), an *improbatio* (disproof), and a *solutio* (solution). These are intended to function respectively as i) a ground for the truth of what the sophism states, ii) a ground for the falsity of what the sophism states, and iii) a way of determining what the sophism really states and a resolution of the problem engendered by the sophism — often by making a distinction with regard to the *probatio* or *improbatio* — in such a way as to allow both *probatio* and *improbatio* to be true, but one of them with a certain qualification or qualifications added.

In the *improbatio* of the sophism, Burleigh contends that:

Improbatur sic. Omne quod potest esse verum, est possibile; sed impossibile potest esse verum, igitur impossibile est possibile. Conclusio falsa, ergo aliqua praemissarum; non minor, ergo maior […] Ad improbationem dico, accipiendo maiorem

secundum quod vera est, non sequitur conclusio haec: 'Impossibile est possibile'[3], sed ista: 'Quod potest esse possibile est impossibile', et hoc est verum[4].

[The sophism] is disproved thus. Everything which can be true, is possible; but the impossible can be true; therefore, the impossible is possible. The conclusion is false, therefore, [so is] one of the premises. [But] the minor [premise is] not [false]. Therefore, the major [premise is]… For the disproof, I claim (accepting the major [premise] according to that which is true) this conclusion, "The impossible is possible" does not follow. But this [conclusion], "That which can be possible is impossible" (and this is true) does [follow].

In claiming that "That which can be possible is impossible" is true, Burleigh explicitly endorses (2). Stated formally, this amounts to:

(2') $MM\alpha \bullet \sim M\alpha$

Again, it is easy to show formally that this entails a denial of one of the characteristic axiom schemata – this time the S4 axiom schema[5]:

{1}	1. $MM\alpha \bullet \sim M\alpha$	assumption
{1}	2. $\sim(MM\alpha \supset M\alpha)$	negation of conditional
		Q. E. D.

If I have correctly interpreted Burleigh's two propositions, evidently the denial of these two characteristic axiom schemata follows. It remains to give some account of why Burleigh makes this move.

3. The meaninglessness of modal iteration

Unfortunately, my interpretation of Burleigh's rejection of these modal axiom schemata runs into a bit of a problem when I seek to account for his motivation of that rejection. Normally, when one accepts or rejects a modal axiom schema one does so upon compelling logical or ontological grounds. Does Burleigh have such a reason, based on his expressed views, for rejecting these schemata? It seems so. Notice that both axiom schemata require an iteration of a modal operator. Hence, if one explicitly rejects a meaningful iteration of such operators, one seems forced to deny the meaningfulness of these axiom schemata. And Burleigh seems to deny the meaningfulness of such iteration. He argues:

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[3] Reading 'possibile' for 'impossibile'.

[4] W. BURLEIGH, o.c., pp. 240-1.

[5] For convenience's sake I utilize the S4 axiom schema stated in terms of possibility, viz., $MM\alpha \supset M\alpha$, rather than the more usual statement in terms of necessity, viz., $L\alpha \supset LL\alpha$. It is easy to verify, however, that these formulations are equipollent.

Propositio modalis non potest modificari per modum alicuius, quia sic essent duo modi. Et tunc, si sit idem modus, erit nugatio. Si diversi modi, vel erit oppositum in adiecto vel erit nugatio, ut si unus modus sit in plus quam alius, sicut possibile est in plus quam necessarium[6].

A modal proposition cannot be modified through the mode of anything, because then there would be two modes. And then, if it were the same mode, it would be redundant; if different modes, either it would be opposite in adjacency, or it would be redundant, as if one mode is useless compared to another [mode], as the possible is useless compared to the necessary.

This clever argument directed at the meaningfulness of iterated modal propositions is astonishing, especially when one considers that the very propositions Burleigh wishes to defend in the sophism above are just such propositions. Clearly, there must be a tension in Burleigh between these opposing views. How can one best relieve that tension? I shall examine two possible ways out for Burleigh.

4. An attempted harmonization of Burleigh's opposing views

First, one could argue that (1'') and (2') would be better expressed with tense operators included in the formalization, so that, for example, the pre-formalized version, (1'), would amount to, roughly:

(1''') "What is impossible now, will later become possible"

or vice versa. Similarly, (2) might be expressed roughly (again, prior to formalization) as:

(2'') "That which is possible at one time is impossible at another time".

In this way the first possibility operator in the formal version (2') would stand for a metalogical predicate, indicating that something can be possible at one time but impossible at another. On this interpretation, it is not clear that (1') and (2), by themselves, entail the denial of the axiom schemata of S5 and S4. (Of course, the denial of the meaningfulness of these axiom schemata would still follow, however, from his denial of the meaningfulness of modal iteration in general.)

But there is a problem with this interpretation. The wording of the Latin text seems to preclude (1''') as an accurate interpretation of Burleigh's (1'), for in (1'''), the scope of the 'potest esse' seems not to include the 'impossibile', and hence seems to rule out its being used to modify it. But it seems clear in (1') that Burleigh intends precisely that the modal phrase 'potest esse' should operate on the modal word 'impossibile'. Consequently the tension in Burleigh's views remains.

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[6] W. BURLEIGH, o.c., p. 236.

A second, and I believe better, try would be to consider (1') and (2) as modal characterizations of assertoric propositions — even though such sentences contain modal words. Burleigh claims that:

Adhuc necessitas vel contingentia potest accipi dupliciter, uno modo ut res, alio modo ut modus. Quando enim modus est subiectum vel praedicatum in propositione, tunc accipitur ut res; sed quando est determinatio compositionis, tunc accipitur ut modus; sic enim dicendo: 'Hominem esse animal est necessarium', secundum quod li 'necessarium' praedicatur, accipitur ut res, et non facit propositionem modalem, sed tunc est propositio de inesse. Sed quando necessitas vel contingentia determinat compositionem inhaerentiae preadicati ad subiectum, tunc est propositio modalis, ut: 'Homo necessario est animal', 'Homo contingenter est albus'. Et tunc modus non est praedicatum nec pars praedicati nec est dispositio praedicati, nec etiam est subiectum nec pars subiecti, nec etiam dispositio subiecti, sed est dispositio compositionis principalis unientis praedicatum cum subiecto[7].

Necessity or contingency can still be taken in two [further] ways, in one way [it can be taken] as a thing, [but] in another way [it can be taken] as a mode. For when the subject or predicate in a proposition is a mode, then [the modal word] is taken as a thing. But when [the modal word] is a determination of the composition, then [the modal word] is taken as a mode. For by saying thus: "That man is an animal is necessary", according to which 'necessary' is predicated, [the modal word] is taken as a thing, and it does not make the proposition modal, but in that case it is an assertoric proposition. But when necessity or contingency determines the composition of the inherence of the predicate to the subject, then it is a modal proposition, as: "Man necessarily is an animal". But then the mode is not a predicate, nor a part of the predicate, nor is it a disposition of the predicate, nor is it even the subject nor a part of the subject, nor even a disposition of the subject, but [instead] it is the disposition of the main composition uniting the predicate with the subject.

Given the view above, Burleigh might allow (1') and (2) to be meaningful[8]. So one would have to read (2), for example, as:

(2''') "It is possible that something[9] is possible, but is in fact impossible".

On Burleigh's view, (2''') would contain but one modal word as a 'thing' and the remainder of the proposition would be assertoric[10]. One could then argue that the characteristic axiom schemata of S4 and S5 would still be disallowed by Burleigh — not because his propositions entail their denial but because these schemata are usually expressed as iterated adverbial modal propositions: 'necessarily necessary', 'necessarily possible', and so on, and these — insofar as they represent genuine modali-

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[7] *Ibidem*, pp. 234-5.
[8] *Ibidem*.
[9] The "thing" in question here must almost certainly be a proposition if (2''') is to make any sense.
[10] W. BURLEIGH, o.c., pp. 234-5.

ties and not 'things' — are subject to his analysis of modal iteration and consequently are meaningless. On this interpretation, Burleigh could both deny modal iteration — understood as adverbial iteration — and yet hold (1') and (2) as meaningful since they would be non-adverbial modalizations of assertoric propositions, and hence they would not really be modal at all, but rather the modes would be 'things'.

This solution, however, has its flaw, for in contemporary logical theory, (2''') would be properly formalized by (2'), and would not differ from the claim that some sentence is both possibly possible, and impossible. And in that case it is not clear how one ought to formalize these propositions to reflect the difference in adverbial and non-adverbial modes, since there seems to be no such contemporary distinction in formal logic. Put simply, if one lets the first modal operator, in a pair of modal operators, stand for a metalogical remark about a sentence containing a modal word, the formalized result would be the same as if one applied an adverbial modal word to an adjectival or nominal modal word. To take a simple example, suppose I were to claim that:

(3) "2 + 2 = 4 is necessary".

Now suppose I claim further that the truth of (3) holds only in light of God's will and that once God wills something it is eternally true, and what is eternally true is necessary. So, I could contend that:

(3') "It is necessary that (3) be true".

But this seems to me to amount to the same claim, logically, as:

(4) "It is necessarily necessary that 2 + 2 = 4".

And if (3') is logically equivalent to (4), Burleigh may not be able to resolve the tension in his claims in the manner I have suggested, even if he thinks he can.

In summary, then, Burleigh affirms two propositions which entail the denial of the S4 and S5 axiom schemata, while denying the meaningfulness of modal iteration. Since the propositions he affirms rely precisely on such iteration, a tension develops between these views. In relieving that tension, Burleigh may have recourse to a distinction between adverbial and non-adverbial modalities (which, in my considered opinion, is surely his actual "solution" to the problem), but then it is not clear how he can avoid this distinction from collapsing, leaving unresolved the tension in his position[11].