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Gnoseology, Ontology, and the Arrow of Time

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1. The Philosophical Aspect (J.J. Sanguineti)

1.1. The asymmetry of time

The present article on the 'arrow of time' is divided into two heterogeneous sections (the reader may choose which one to read first). In this philosophical section I intend to present and comment upon the scientific section written by Castagnino, a physicist who has worked especially in quantum mechanics and quantum cosmological models. I will try to provide a philosophical comprehension on the topic, which implies its introduction in the area of philosophy of nature and philosophy of science. A collaboration of this sort is indispensable for certain speculative problems on the nature of the physical world. Science has not the aim of philosophy, but it gives some indications that cannot be overlooked by the philosopher of nature. I hope that the reader in the following pages will understand in a practical way the need of a mutual relationship between physics and philosophy.

The problem of the direction of time in the physical world is just one aspect of the general problem of time. It is usually agreed that time, imagined as a line, has a direction (like an *arrow*, according to the famous Eddington expression), in the sense

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that it goes from the past to the future and not the other way round. The feeling that this is the right and necessary direction of time is related to the impossibility of making trips to the past, or of stopping the flow of time, and it is incompatible with the idea that time is a subjective illusion[1].

But what does 'going from the past to the future' mean? Since time is in some way reducible to movement and to any kind of alteration, though it is not exactly equal to them, that expression means at least that some physical evolutions are naturally irreversible. They follow some path and cannot travel in the opposite direction. A ball can go from A to B and then turn back from B to A, but we have never seen in nature the inversion of processes like the burning of a sheet transformed in ashes, the breaking of a glass, and so on.

Notice that these examples regard a passage from order to disorder (in coincidence with the second principle of thermodynamics), or from unstable physical states which *spontaneously* (without any additional cause) tend to an equilibrium state. Our article deals exclusively with this aspect of the arrow. The direction of time could also mean a passage from disorder to order, as it happens in evolutionary processes (but every acquisition of order spends energy and so it obeys the second principle as well). Furthermore, if the upcoming order is actually new, not produced by a previous law, then we have a kind of creation, which in human affairs transforms time in *history* (the historical future is not written in the past).

The future, then, can bring on order or disorder, lawfully or not. Some future events are repeated, at least in certain features. This implies that we go back to aspects already seen in the past. A total repetition of everything would amount to a real return into the past.

We are concerned with what strangely appears as a *law of time:* events universally do follow a certain direction towards equilibrium (with less order), i. e. to a more elementary and more stable order. Generally, the physical laws have nothing to do with the direction of time. They describe a behavior remaining identical if we change the direction of time, that is, if we imagine the same process as an inverted film. Time in physics appears to be, like space, without special directions or perfectly symmetric. The great exception to this symmetry was, since the nineteenth century, the second principle of thermodynamics, which apparently showed a natural preference for some special direction, creating a difference in it which deserves the name of *future* and *past*. But thermodynamics deals with energetic processes and then the problem becomes universal, since energy is involved in any natural process. What it is at stake here is the nature of the physical universe. Is it dominated by time or by an eternal law?

[1] On the problem of the arrow of time, see H. Reichenbach, *The Direction of Time*, University of California Press, Berkeley and Los Angeles 1956; H. B. Hollinger, M. J. Zenzen, *The Nature of Irreversibility*, Reidel, Dordrecht 1985; P. Kroes, *Time: Its Structure and Role in Physical Theories*, Reidel, Dordrecht 1985; P. Horwich, *Asymmetries in Time*, MIT Press, Cambridge (Mass.) 1987; P. Coveney, R. Highfield, *La freccia del tempo*, Rizzoli, Milano 1991; H. D. Zeh, *The Direction of Time*, Springer, Berlin 1992; S. Savitt (ed.), *Time's Arrow Today*, Cambridge University Press, Cambridge 1995; J. J. Halliwell et al. (ed.), *Physical Origins of Time Asymmetry*, Cambridge University Press, Cambridge 1996.

We will restrict the discussion to physics, without entering the domain of biology or anthropology. The asymmetry of time may be observed and discussed in different fields of modern physics. Since it seemingly reveals a fundamental trait of the world, it is very relevant to philosophy of nature. Any difference raises the question of its cause. In a rationalist approach, it is more natural to be satisfied with symmetry. A non eternal law is not a perfect law. If the law changes, we are entitled to ask why, looking for a higher law[2]. The course of events assuming a *special* direction looks more like a *fact* than a law, that is, something 'happens to be' and so it is expected to be explained by a superior law. In the late nineteenth century Boltzmann tried to reduce the aforementioned second principle to microphysics (statistical mechanics). He concluded that the principle was only probable, since nothing in theory prevented the particles to converge in an ordered movement, producing macrophysical events (mostly improbable) such as the spontaneous reordering of a destroyed building. The debate was never satisfactorily concluded[3].

If within microphysics there would be no temporal direction, but notwithstanding the arrow does appear at the phenomenological level, the explanation could be that we observers see an apparent direction in the surface simply because we cannot measure every particle with an absolute and deep precision. Indeed, the statistical character of physics stems from the fact that we cannot deal with every particle but must content ourselves with a global approach regarding collections.

Hence the idea that the arrow of time in physics is due merely to the process of an imperfect measurement, which is the thesis of the 'gnoseological school' mentioned by Castagnino, opposed to the 'ontological school', according to which the direction of time is real (*school* means here 'theoretical position'). Boltzmann, among others, represents the gnoseological approach and Prigogine, in our times, is the most famous supporter of the ontological character of the anisotropy of time[4]. If the distinction of past and future should depend only on our observational and anthropomorphic approach, then the temporal directions would be analogous to the spatial directions of up and down, which are such only from our bodily perspective[5].

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[2] It is possible that «principles that we *now* regard as universal laws will eventually turn out to represent historical accidents» (S. Weinberg, *Dreams of a final Theory*, Pantheon Books, N. York 1992, p. 38).

[3] See E. Bellone, *I nomi del tempo*, Boringhieri, Torino 1989.

[4] «We are becoming more and more conscious of the fact that on all levels, from elementary particles to cosmology, randomness and irreversibility play an ever-increasing role. *Science is rediscovering time*» (I. Prigogine and I. Stengers, *Order out from Chaos*, Collins, Glasgow 1988, xxviii). «The view that irreversibility is an illusion has been very influential and many scientists have tried to tie this illusion to mathematical procedures, such as coarse graining» (I. Prigogine, *From Being to Becoming*, Freeman, N. York 1980, p. 12). See below the role of coarse graining in the scientific section.

[5] For Einstein, the divisions of time were local perspectives. «Per noi che crediamo nella fisica, la divisione tra passato, presente e futuro ha solo il valore di un'ostinata illusione» (A. Einstein, Letter to Besso's son and daughter, March 21, 1955, in *Albert Einstein. Opere scelte*, a cura di E. Bellone, Boringhieri, Torino 1988, p. 707). In this sense, the gnoseological school may imply an ontology as well: the universe is all in act (Parmenides).

1.2. Castagnino's paper on time

I turn now to the scientific treatment, in order to give a more qualitative version of it which, I hope, may facilitate the reader's insight into the philosophical core of the problem. This second section is partly technical, making use of mathematical language that renders the exposition precise and scientific. But it is obviously related to problems of philosophy of science and philosophy of nature. For the philosopher, it is likewise a very perspicuous example of the way in which science accomplishes its task (it shows the measurement approach of physics and the recourse to mathematical devices such as spaces).

Since the controversy on the direction of time is related to observations recorded by physical instruments, the first part of Castagnino's paper regards measurement (n. 2 and 3). He presents a general theory of measurement which, in modern physics, cannot be but statistical. This theoretical framework has actually arisen in statistical classical mechanics but is now generalised in quantum mechanics. In this sense it can deal with any physical event of the universe, with a certain approximation and in probabilistic terms.

Modern physics is concerned with the description of dynamic evolutions of systems. The universe is the last system, supposed isolated since by definition there is nothing outside it. A set of elements belonging to a system (e. g., points, mass-points, etc.), on having certain geometrical properties, are represented in *spaces*. This term does not refer to the ordinary space of our common perception, but it is a mathematical construction thought of to describe, through a selection of features, sets of things and their evolution (represented as lines, surfaces and so on within the selected space). It is obvious that we are dealing with *entia rationis* with a foundation *in re*. The space here is like a window open to the world, with all the limitations of a window (a partial view).

The physical description lies on some *observables* i. e. data as seen by the instruments of measurement. The evolution of the observables produces different *states* of the system. They are ruled by the *equations* (laws), which state the invariant evolving of the systems according to some parameters. On assigning specific numerical values to the variables of the equation (e. g. the initial conditions), we obtain its *solution*.

The measure of different observables in a state of the system allows, then, to measure that state, and even to measure, within limits, the physical state of the universe (in quantum gravity cosmologies), which is the great system wherein every other one is a subsystem. The result of the measurement of each observable is expressed in terms of a density $p(x)$, related to a continuous variable, whose values are taken e. g. for some subintervals [0, 1] of a length x.

Different *qualities of measures* are considered, ranging from less to more and more precision. All these different qualities (Hilbert, coarse-graining, Schwarz, and Hardy) are associated to *different kinds of spaces*: Hilbert space H, coarse-graining space C, etc. The name *coarse-graining* comes from the precision with which the space is divided in 'grains', like a photograph. This is the typical method employed by statistical mechanics (e. g. for the description of a gas). Each space is useful for some purposes within particular sections in physics, and they are related to each other by logical inclusion. Beside the space built up from sets of observables (*space of the observables*, symbolised as *O*), there is the corresponding *space of states S*, which is a function of the former. Notice that the space of the observables (i. e. the world as seen by some branch of positive science) is the material or sensible basis of the scientific network. The space of the states serves to describe the physical evolution in this world.

The following step copes with the time arrow in the *mixing systems* (n. 4). They belong to chaotic systems, whose dynamic behavior is irregular due to their degree of complexity (they correspond to the real world better than the simple non mixing systems considered in old classical mechanics). A drop of blue ink diffusing in a glass of water (a case studied by Gibbs) is a mixing system. Its volume remains the same but it is homogeneously distributed throughout the water, ending up in an equilibrium state. The opposite process does not occur in nature. It may be considered as possible in theory, but it is 'non physical', or 'not physically allowed'. For comparison Castagnino uses the famous theoretical example of the *baker's transformation*, in which a quantity of low quality flour (analogous to the ink drop) is distributed again and again in a bread dough, with the technique of cutting and joining again the dough many times so as to reduce the low quality flour to ever thinner and thinner filaments. Remaining the same, this flour, at the end, occupies homogeneously the whole dough.

The evolution in mixing systems from non-equilibrium to equilibrium is the motion we observe towards *the future*. Its spontaneous inversion, from equilibrium to non-equilibrium, is never seen in nature: an ink drop does not come out spontaneously or naturally from a diffused state in the past (notice that these words here, *spontaneous* and *natural*, are crucial for the philosopher of nature). From the situation of the ink drop concentrated in a point in the water (time=0), if the drop behaves like the flour in the baker's transformation, we can go in theory to the future or to the past, to find out the diffused state of the ink. The film with the ink diffusing (going from t=0 to the future) or, inversely, with the diffusion forming an ink drop (going back from t=0 to the past) is the same for the physical laws (and both processes can be seen one after the other in the theoretical, not real, baker's transformation). But only the direction towards the future is *really* seen in nature[6]. This is precisely the problem of the arrow of time. In the following scheme it is easy to visualise what we are saying (some additions help to understand what will be said later on):

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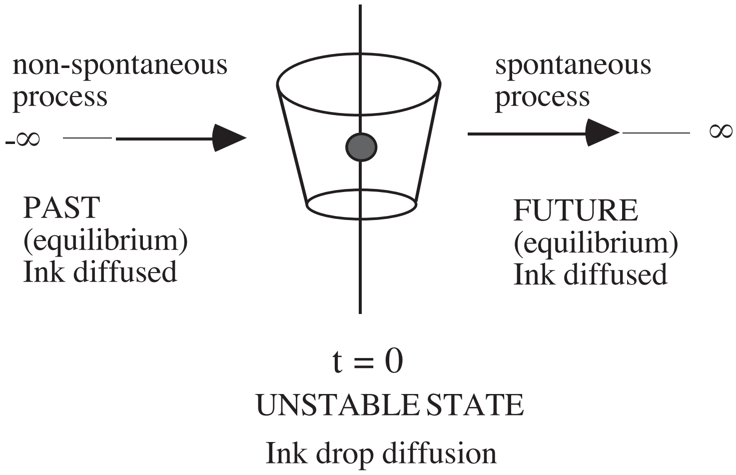
[6] The expression "going to the past" here is purely epistemological. Of course, the inverted process of $0 \to \infty$ is $\infty \to 0$. Presupposed here is a difference between the "physical time arrow" and the "psychological time arrow" (the later is involved in the notion of *observation*). If we have never seen a spontaneous forming of an ink drop from a diffused state, this means that there is a correspondence between the psychological and the physical orientation of the course of events. In a realist philosophy, observation is accomplished in the psychological present, but it implies a participation in the real course of events.



Castagnino shows two mathematical ways of coping with the process. The 'gnoseological school' (see n. 4.1) uses the *coarse-graining* technique (spaces with different degrees of coarse graining: the notion of 'graining the space' is, obviously, relative to the observer). The school makes use of the corresponding *C space*, the kind of space used in coarse-graining procedures. The evolution of mixing processes can be explained with this device. It could be said, then, that we see the evolution towards equilibrium at the infinite future or in the period $t \longrightarrow \infty$ (the ink diffusing) because precision is lacking. This amounts to say that, in the case of an infinite precision, there would be no arrow of time. Entropy would be merely a lack of information. I presume that this position is tied to determinism. If everything is determined, it makes no sense appealing to a difference between past and future. There would be a mere mechanism and its reversal is perfectly conceivable.

The space *C* of the coarse graining method is time symmetric and it is unable to describe the breaking of time symmetry. It cannot deal with a world with time symmetry broken, or with the non physical period $t \longrightarrow -\infty$ (time reversal: going to the past). It simply ignores this period.

The 'ontological school' (n. 4.2) should use a more accurate observer space (Schwarz space), whose mathematical properties allow to see the evolution towards the future and towards the past in a *Dirac's comb* (this device includes a representation of both temporal directions: the 'horizontal comb' is the equilibrium in the far future, while the 'vertical comb' is the equilibrium in the far past). The task is now to choose one of them, namely S_- (observer space turned up to the future), with its corresponding state space S^x, since this choice fits with the real evolution perceived in nature. This passage is argued in n. 5. Since the necessary condition of not changing the space of the observables is not satisfied in the past direction (where 'non physical events' happen), but only towards the future, the recourse to a specific Hardy space of the observables is required; in symbols: O_- will be H^2 (changing the signs + and - would mean to change the course of time).

This is not the complete solution of the problem of the time arrow, of course, being rather a mathematical procedure of dealing with time that more clearly reveals what it is supposed to be an ontological property of nature. The strategy is remarkable for a philosopher of science. It shows how the scientist can choose mathematical instruments which allow scientific constructions to (partially) manifest what nature is. Undoubtedly, the measure of time is likewise related to man, the author of measure. Physical time, as seen by science, corresponds to real nature, but it is an elaboration of human mind as well. Indeed, the arrow of time is perceived from the *spaces of the observer*, and that is why Castagnino's main thesis in this paper is that the arrow of time is both ontological and gnoseological, since the image of the universe depends on reality but also on the observer. Our knowledge, as Aquinas would say, corresponds to the *modus essendi* but also the *modus cognoscendi*[7]. There's a nice convergence here between philosophy and science.

The last section (n. 6) is much closer to philosophy. The assumption of a universal arrow of time, founded on a never contradicted physical experience, involves the transference to a cosmological model. The simplest one could be the universal global system proposed by one of the most important contemporary philosophers of time close to the scientific area, Hans Reichenbach[8]. The *global system*, adapted here for the purpose of the paper, assumes that every branch subsystem begins in a non-equilibrium state, evolving towards equilibrium (increasing entropy). This branch subsystem, though relatively isolated, has acquired its improbable initial energetic state from a previous branch subsystem, and so on. Castagnino relates the process of the ink diffused in the water to the origin of the elements and particles, ultimately going back to the origin of the universe. The cosmological global model is adequate even for the quantum level (microphysics), if we take into account the scattering processes. These processes involve the creation of *unstable quantum states* (unstableness is the key of irreversible processes and, in general, of the very idea of a cosmic evolution). The unstable states last either for a brief or a long time or, in other words, they cause a delay, therefore they produce time. At the end, they decay into a stable state, where almost nothing occurs.

The arrival to an unstable state is a creative process which ordinarily requires a preceding source of energy. On the contrary, the consequent decay is natural. The scattering processes are here analogous to the formation of the ink drop and its diffusion. They have a clear energetic cause (e.g. accelerators of particles). A question arises regarding the cause of the first unstable state with which the universe starts its life. If we do not go back further on, it is because there is no past before t=0. Contemporary quantum cosmology postulates this kind of origin, so the arrow of

7 Aquinas, *S. Th.*, I, q. 84, a. 1. Plato's mistake was to assume that forms were *in re* just as they were *in cognoscente*. Therefore, there would be no difference between ontology and epistemology.

8 H. Reichenbach, *The Direction of Time*, cit.

time presents itself as a *cosmological* direction towards the future[9]. But even within classical standards and without the recourse to an initial 'creation of time', it is known that the history of the universe goes back at least to early unstable states.

It could seem then that we are constrained to use the mathematical device of the O_- space of the observer and not its specular image O_+ with the inverted time making the whole difference. But now there is a surprise. Since in a cosmological model *all* the different local arrows of time would have changed at once, going in the opposite sense (including the psychological arrow of the observer, which is a part of the universe), there is no physical difference (in the scientific meaning of *physical*) between the opposite directions of time, just as there are no physical differences between the pure geometrical directions of right and left. So Castagnino's conclusion is that 'we must choose' because we are constrained by the facts and not by the laws. This choice, however, is irrelevant in physics as a science. The choice is supported by what we have ever seen in the real world, not from the nomological physical description.

1.3. A philosophical choice

The difference between the gnoseological and the ontological school is philosophical, since there is no empirical constraint from the scientific point of view to overcome the time-symmetry of the nomological account of physical processes. The philosophical option in favour of the gnoseological school is more akin with the positivist attitude. Positivism leads to speak about nature only in scientific terms. Paradoxically, this approach creates several philosophical problems. In fact, if the difference between past and future is not physical, the temptation arises of assigning it to the situation of the observer (just as the right and left directions do change with the movement of the observer). Giving a special privilege to the observer ends up in dualism in its rationalistic version.

The ontological school's option in favour of the reality of the direction of time is ontological because it acknowledges the existence of a field of reality outside the framework of mathematical physics. Castagnino argues that there are physical processes that 'we have never seen'. This appeal to the facts (in the sense of dismissing theories concerned with facts we have never seen) is not empiricist or positivist, but ontological. I hope the reader will understand this subtlety. Positivism restricts its view to the facts as considered by scientific laws, and in this sense it has no problem to imagine fictitious facts, never seen, but anyway allowed by the laws. An

9 This statement is limited to quantum cosmological models dealing with a 'creation of time' at the very beginning of our 'classical' universe. («There exists an arrow of time only because the universe originated in a less-than-maximum entropy state... The expansion of the universe has caused it to depart from equilibrium»: P. Davies, *Stirring up Trouble*, in J. J. Halliwell et al. (ed.), *Physical Origins of Time Asymmetry*, Cambridge University Press, Cambridge 1996, p. 127). We let aside a quantum gravitational scenario without (classical) time, wherein our universe would emerge with its time. See J. Halliwell, *Quantum Cosmology and Time Asymmetry*, in *ibid*., pp. 369-389, and our article *La creazione nella cosmologia contemporanea*, «Acta Philosophica», 4, 1995, pp. 285-313, for the theological and philosophical problem.

Aristotelian ontology, on the contrary, is built upon reality as such, not upon imaginary or possible reality.

A merit of Castagnino's argument is the stress on the philosophical character of the whole choice. Even the positivistic choice in favour of the purely gnoseological interpretation of the arrow of time is pretty much philosophical, and it creates the very difficult philosophical problem of dualism between reality and observer within the physical description.

The argument that 'we have never seen those facts' involves the coincidence between the psychological and the physical arrow[10] (to see an inverted film is to put the physical arrow in contrast with the psychological arrow). Anthropologically, we should conclude that our time is rooted in nature. The fictitious inversion of natural time would imply the independence of our psychological arrow from nature. Descartes would be right against Aristotle. Of course, we can 'think' of the reversal of a process (since human thought is independent from space-time), but that thought exists within the real psycho-physical time through which we participate, as physical observers, in the ontological display of our world.

The points I have commented upon show to which extent some choices of mathematical instruments in physics may be conditioned by philosophical motivations. A mathematical reading of reality, it is frequently said, is blind to the natural traits of reality. However, the many different mathematical devices used in the natural sciences may help our mind to get an insight into the ontological structure of reality. Philosophy is not science, but a philosophical view is not impossible on the basis of physics[11].

2. The Physical Aspects (M. Castagnino)

2.1. Introduction

For those among scientists that believe that *Truth* must be found in *Science* as a whole, and not in any isolated chapter in science, the present situation is highly discouraging. Scientists are so specialized that they ignore completely what is happening in the neighboring fields of their own speciality. Precisely, philosophers cannot understand physics, because it is written in mathematical language, and physicists can not understand philosophy, because it is *not* written in mathematical language. The author of this second part is not free from this problem because, with the exception of a few concepts, he ignores philosophy. Nevertheless, this paper is a modest attempt to solve this problem, trying to find a physical-philosophical answer to one of the most important questions of modern physics: the problem of the *arrow of time* [1][12]. Even if this paper is addressed to philosophers, some high school formulae are

[10] Recall Reichenbach's question: «Why is the flow of psychological time identical with the direction of increasing entropy?» (*The Direction of Time*, cit., p. 269).

[11] The authors are collaborating in a book which will develop the problem of time both in physics and philosophy.

[12] For numbers [1], [2], [3], etc. see the *References* at the end of the paper.

used, since they are unavoidable. Also, some explanations in the footnotes are devoted to physicists and mathematicians and they can be neglected by philosophical readers.

The image of the universe depends on the universe itself and on the observer that looks at the universe and sees its image. This seems an undeniable statement. The image of the universe is essential, since we can only understand the universe through the images obtained by the observers. Then in the image of the universe there are two components:

I.- The universe itself, namely its ontological nature.

II.- The observer that looks at the universe, namely the knowledge or information that the observer obtains when he studies the universe.

This idea is so convincing that we can extend it from the whole universe to any part or feature of it: 'the image of any feature of the universe depends on the feature itself and on the observer that studies this feature'. We will postulate that this is true for almost any feature of the universe.

One of the reasons of this paper is precisely to show that this idea is not accepted by many physicists, and we will demonstrate this fact using the arrow of time, one of the features of the universe, as an example.

The problem of the existence of the arrow of time or, what is the same thing, the problem of time asymmetry of the universe, can be formulated asking the following question [2]:

How can it be that the universe is time-asymmetric if all the relevant physical laws are time-symmetric?

In fact, the main laws of physics, the Newton laws of mechanics, the Maxwell equations of electromagnetism, the Einstein equations of relativity, the Schrödinger equation of quantum mechanics are time-symmetric[13]. Nevertheless the universe has several time asymmetries, namely the various arrows of time: thermodynamic (entropy grows towards the future), electromagnetic (we use retarded solutions), psychological (we feel that the past is different than the future), etc., that must be explained. Even if these asymmetries are not contained in the physical laws themselves, they nevertheless belong to the object under study: the universe. Then we can say that these asymmetries are not 'legal'; they are 'factual' or 'objective', since they are asymmetries of the object, but not of the laws that rule the object. Now the asymmetries of the object can have a gnoseological or an ontological origin. Therefore, to solve the problem, the physicists are divided into two schools:

1.- The gnoseological school (Boltzmann [4], Zwanzig [5], Zurek [6]). This school explains the arrow of time saying that it is created by the act of observation, i. e. by the observer[14]. Precisely, for this school there is a microscopic universe that is

[13] We will neglect the time-asymmetric laws of weak interactions, since it is very difficult to imagine a mechanism that explains the time asymmetry of the universe based on these laws [3].

[14] This definition of the gnoseological school is necessarily schematic, since traces of the ontological argumentation can be found in its authors.

time symmetric and where there are no arrows of time, and there is a macroscopic universe, the one that macroscopic observers see, where there is one or many arrows of time, created by our inability to measure the microscopic universe with infinite accuracy. There is no ontological reason for time asymmetry, according to this school, since the real universe, the microscopic one, is essentially time symmetric[15].

2.- The ontological school. This school states that time asymmetry is an ontological characteristic of the universe[16]. In its extreme version this school refuses any gnoseological explanation, since it forbids any reference to the measurement procedures to explain the arrow of time (Prigogine and co-workers [9]).

We will try to demonstrate that both schools are wrong (even if they are partially right), because their reasoning is incomplete and the common sense statement of the beginning of this section, 'the image of any feature of the universe depends on the feature itself and on the observer that studies the feature', is the clue to solve the problem. Thus we will try to prove that the arrow of time depends on the universe and it can be seen only if the observer uses an adequate measurement apparatus.

2.2. Observables and states

Let $\mathcal{O}$ be the set (or space, namely a set endowed with certain mathematical properties) of all the observables (namely all the observation apparatuses, e. g. the apparatus that measures the distance to a certain star) that we will use and $\mathcal{S}$ the set (or space) of all possible states of the universe (e. g. the state of the universe today)[17]. If $\mathrm{O} \in \mathcal{O}$ is any observable and $\rho \in \mathcal{S}$ is any state, a *measurement* is made with the observable O in the state ρ with a result m (e. g. the distance to the star today). It is clear that if we measure all the observables of $\mathcal{O}$ in a state ρ (precisely all the distance to all the stars today, the mass of all the stars today, the temperature of all the stars today, etc.), we do know all the data about ρ and, in this sense, we know the state of the universe ρ (i. e. the state of the universe today). It must also be clear that these definitions are completely theoretical, since we are referring to measurements done with infinite precision, and these measurements are impossible. Real measurements are always affected by some errors. To introduce these errors systematically we can consider that the universe is only known in a statistical way. So to continue we must add two important components:

1. Statistics. We know that really modern physics has proved that we cannot speak about the occurrence of facts but only about the probability of the occurrence of these facts. Nowadays physics is essentially a statistical science [10]. So the result

[15] The line of thought that explains time asymmetry introducing stochastic noises can be considered a variation of this school [7].

[16] It depends on the solution of the physical laws that describes the present universe and, since we know the mathematical equations obtained from the physical laws, it depends on the initial conditions of the present universe [8].

[17] Really our 'universe' can be any closed isolated system within the real universe, and nothing will change below, since essentially the universe is just a closed isolated system. Nevertheless we will continue to talk about the 'universe' (also because to obtain a complete isolation of a subsystem of the universe is merely a theoretical fact).

of the measurement is not just *m* but a set of possible results m_1, m_2, where we can only tell the probability of each result p_1, p_2,... So the best we can have is the *mean value* or *weighed average* of the measurement, precisely

$$< O >_\rho = \frac{m_1 p_1 + m_2 p_2 + ...}{p_1 + p_2 + ...}$$

and since the sum of the probabilities is the probability 1 (the certitude), namely $p_1 + p_2 ... = 1$, we obtain:

$$< O >_\rho = m_1 p_1 + m_2 p_2 + ... = \sum_i m_i p_i \qquad (1)$$

where the last symbol is a sum, necessary to avoid the repetition of similar terms like $m_1\ p_1$, $m_2\ p_2$, etc., which we generically call $m_i p_i$, being *i* a generic index. So the symbol Σ_i means 'add all the similar terms $m_i p_i$ '.

2. Continuous nature of the measurements. Generally the possible measurements are not a finite or discrete set like m_1, m_2,... but a continuous one *m(x)* where instead of the index *i* we have the continuous variable *x* (a *discrete* set of points is an infinite set of isolated points; in a *continuous* set the points are not isolated, but they belong to a continuous curve, surface, volume, etc.). In fact, let us consider an usual measurement apparatus (a barometer, a thermometer...). *x* would be the coordinate of the position of the hand of the barometer (eventually an angle), or the position of the mercury scale in the thermometer which in fact is continuous, and *m* the number written at each position (*m* HPascal, *m* degree centigrade...). Then *m(x)* is the function that relates the position of the hand with the measured quantity. To simplify let us suppose that the possible positions of the hand are the points of an interval, let us say the interval [0,1]. So, in this continuous case, m_i is transformed in $m(x)$,$[0 \leq x \leq 1]$. Then, what happens with the probability p_i?

Each time we deal with continuous objects we are forced to define densities. Let us imagine a discrete set of points P_1, P_2,... with masses M_1, M_2,... that we consider they are concentrated in each point. The total mass of the set of points is $M = \Sigma_i M_i$. Let us now consider a continuous object. It is impossible to know the mass of each point P since the volume of the point is zero. So we take a small volume ΔV around the point P, we measure the mass contained in it, ΔM, and we define the density $\delta = \frac{\Delta M}{\Delta V}$. Then $\Delta M = \delta \Delta V$ and the total mass of the body is $M = \Sigma \delta \Delta V$, where the sum is extended to all the small volume in which we have divided the body.

In the same way we will define the probability density *p(x)*. For every small subinterval of [0,1], of length Δx, there is a probability Δp and a density of probability: $p(x) = \frac{\Delta p}{\Delta x}$, in such a way that the probability Δp in the subinterval of length Δx is $\Delta p = p(x)\Delta x$. If the measurement at *x* is *m(x)*, the generalization of the average (1) reads:

$$< O >_\rho = \sum m(x)p(x)\Delta x \qquad (2)$$

where the sum is extended to all the subintervals in which we have divided [0,1][18]. The function *m(x)* (the set of possible measurements) is a characteristic of the observable O, while the function *p(x)* (the probability density of each measurement) is the probabilistic definition of the state ρ. If we know the probability of each possible measurement of a state of the universe, we know the probabilistic state of the universe. This is the statistical translation of the previous definition (see the beginning of n. 2), when we use infinitely exact measurements: 'It is clear that if we measure all the observables of *O* of an state ρ (...) we do know all the data about ρ and, in this sense, we know the state of the universe ρ'.

Let us finish this section saying that, even if we were referring to the classical level, all what we have said can be rephrased in the quantum level. This will be also the case for all the reasoning below.

2.3. The properties of function *m(x)* and the quality of the observables

Now we reach the central point of the paper: the mathematical properties of the characteristic function of the observables *m(x)*. We will see that the properties of the observables, used by the two schools, are different and therefore these properties are the basis to define and study both schools.

The mathematical properties of the function *m(x)* will define the *quality* of the observable O. Heuristically, if the function *m(x)* is defined in a fuzzy way, we will say that the quality of the observable is bad. On the other hand, if it is defined in a precise way, we will say that the quality of the observable is good. Consider again the barometers hand and the number of HPascal written in the scale. If the positions of the hand are correlated in a fuzzy way with the numbers of the scale, clearly the barometer is of bad quality. If they are correlated in a precise way, the quality of the barometer is good. As we have said, for simplicity, we will consider that the index x takes only values between zero and one, [i. e. $0 \le x \le 1$]. We will give several examples of decreasing fussiness of the curve *m(x)* and therefore several examples of growing quality:

1.- Hilbert quality. The function *m(x)* is a square integrable. To give an intuitive idea of this kind of functions we can consider that they are continuous functions (like the curves we can draw in a paper with a pencil) where, in a discrete number of points, the values of the function are different than those corresponding to the continuous one. I. e. discrete number of points are subtracted from the continuous func-

[18] To make the sum as refined as possible we take $\Delta x \to 0$, i. e. we make Δx as small as possible. Then, in the limit of infinitesimal interval the mathematician would say that the summatory of the last equation must be substituted by an integral, namely the mathematical generalization of sum to the case where the number of addends is infinite. Then we obtain:

$$< O >_\rho = \lim_{\Delta_x \to 0} \sum m(x)p(x)\Delta x = \int_0^1 m(x)p(x)dx$$

tion and they are elsewhere (fig. 1)[19]. The function can also have a stair shape with jumps or steps (like figure 1´ and also as the curves of the next quality), but for simplicity, let us keep in mind the image of fig. 1. As we will see, Hilbert quality is a very bad one since the curves of figs. 1 and 1´ are quite fuzzy. We will call the space of these functions the Hilbert space *H*.

2.- Coarse-graining quality (we will explain the origin of the name 'coarse-graining' in the next section). Let us divide the interval [0,1] in subintervals $[0,x_1].[x_1, x_2],... [x_n, 1]$ ant let *m(x)* take continuous values in each subinterval. The function *m(x)* looks like a stair, with curve steps, going up and down (fig. 2), with jumps in the points x_i, when we pass from one step to another one. This curve is not so fuzzy as the previous one. Therefore the quality is improved, as we will see below in a less intuitive way. We will call the space of these functions the 'coarse-graining' space *C*. The gnoseological school uses this quality, because it is sufficient to prove a large set of important results, the growing of entropy, the natural tendency to equilibrium, etc. We will discuss this point further in the next section.

3.- Schwarz quality. The function *m(x)* is continuous, differentiable to any order, and square integrable, namely it is a completely smooth curve endowed with a lot of nice properties (fig. 3). The quality is improved. The corresponding function space is the Schwarz space *S*.

4.- Hardy quality. The function *m(x)* has all the properties of a Schwarz function plus other mathematical properties known as *analyticity in the upper or lower complex half-plane* (namely more involved mathematical properties that we will not explain in detail [11]). This is the finest quality we will consider. The corresponding function space will be called the Hardy space from above, H^2 (which has analytic properties in the upper complex half-plane) or from below, H^2, (corresponding to the lower complex half-plane), respectively[20].

Even if we cannot here explain this quality in all details, we are forced to introduce it, since the ontological school is based on this quality.

The only thing the reader should keep in mind is that there is a hierarchy of qualities and that the higher quality corresponds to a higher number of mathematical properties of the curve *m(x)* that makes it less fuzzy and better defined.

To see how this definition of quality of measurement corresponds to the intuitive notion, let us use as an example the simplest probabilistic distribution. Let us sup-

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[19] In a more precise language a mathematician would say that all the curves, obtained by the continuous one by the subtraction of a discrete number of points, are equivalent and that the space in consideration is the space of the corresponding equivalent classes.

[20] The essential property is that *m(x)* can be expanded as a power series as:

$$m(x) = m_0 + m_1 x + m_2 x^2 + ...$$

and, if the real variable *x* is promoted to a complex one *z*, *m(z)* is an analytic function in the upper complex half-plane, in the case of H^2 , or analytic in the lower half-plane, in the case of H^2 . These properties are used by physicists to deduce the 'dispersion relation' [12] and also the 'fluctuation-dissipation theorem' [7]. In these cases physicists are working with the same basis as the ontological school.

pose that we are sure that in the interval [0,1] we measure a fix number e. g.: $\frac{1}{2}$. This state of knowledge corresponds to what is called the δ or *Dirac's state* ρ_D, defined as the state with a probability function $p_D(x)$, which is zero everywhere, in the interval [0,1], but different from zero at $x = \frac{1}{2}$ (fig. 4), since we know that all the probability is concentrated in $\frac{1}{2}$ [21]. We use this state for two reasons:

I. It is the simplest of all.

II. We will see in the next section that we must add infinite Dirac's states to obtain a 'Dirac's comb', an extremely useful state.

Let us now compute the average (2) for an arbitrary curve *m(x)* in Dirac's state $p_D(x)$. Let us divide the interval [0,1] in small subintervals of equal length Δx. As *p(x)* is zero in all intervals but the one that contains $\frac{1}{2}$, the sum (2) will be reduced to just the addend $m(x)p_D(x)\Delta x$ that contains the coordinate $x = \frac{1}{2}$. Namely:

$$< 0 >_{\rho_D} = m(\frac{1}{2}) p_D(\frac{1}{2}) \Delta x$$

Now we can refine the result making Δx smaller and smaller, a process that we symbolize as $\Delta x \to 0$ (and we can take $p_D(\frac{1}{2})\Delta x = 1$ in such a way that, when $\Delta x \to 0$, $p_D(\frac{1}{2})$ grows up to infinite, representing the infinite concentration of the probability at $x = \frac{1}{2}$).

Then, as the small subinterval always contains $\frac{1}{2}$, the final result of the average (2) will be $< 0 >_{\rho_D} = m(\frac{1}{2})$ [22].

Let us see how this state of the universe is measured by the different qualities of functions, i. e. how the different qualities measure $m(\frac{1}{2})$:

1.- Hilbert quality. If the point $x = \frac{1}{2}$ is not one of the subtracted points from the continuous curve, we know the value $m(\frac{1}{2})$. But in the other case this value is unknown (fig. 5). That is why the Hilbert quality is so low.

2. Coarse-graining quality. If the point $x = \frac{1}{2}$ is not one of the point x_i, we know the value $m(\frac{1}{2})$. In the other case, if $x = \frac{1}{2} = x_i$ and corresponds to the value of the jump between two steps, we do not know the value of $m(\frac{1}{2})$, but we do know that this value is contained between the value m_1 and m_2 of the two steps (fig. 5´). So the quality is improved.

3.- Schwarz quality. Now *m(x)* is a nice curve with no jumps or discontinuities, so we know $m(\frac{1}{2})$ for sure (fig. 5´´). The quality is further improved.

4.- Hardy quality. As we have not explained the mathematical notion of analyticity, we cannot tell why we reach the maximum quality. But as we know that when the properties of the function are more numerous the quality improves, and this is the case for the Hardy quality, we can easily understand that here the quality will be improved aswell.

So we see that as quality grows the value of $m(\frac{1}{2})$ becomes better known.

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21 $p_D(x) = \delta(x - \frac{1}{2})$

22 A mathematician would say: $< 0 >_\rho = \int_0^1 m(x)\delta(x - \frac{1}{2})dx = m(\frac{1}{2})$

We will close this section with some results about the inclusion of the spaces that we have defined. It can be mathematically proved that, as the quality improves, the corresponding space becomes smaller, i. e.:

$$H_{\pm}^{2} \subset S \subset C \subset H \tag{3}$$

This fact corresponds with common sense: more accurate measurement apparatuses are less numerous. Moreover, if we request a new mathematical property to a set of functions, the subset of functions endowed with the property is contained in the original set.

According to the quality of our observable space O, we can measure our state space S better or worse. So, for each quality of the observers space we can measure a different space of states. Thus the space of states we can consider depends on the space of observables we use. Then we will say that the space of states is a *functional space* (or a *dual space*) of the space of observables [14] and we will write this dependence as:

$$S = O^{\times} \tag{4}$$

Let us observe that there is an intimate relation between the spaces O and S, in such a way that it can be proved that if one is time-symmetric the other is also time-symmetric, and if one is time-asymmetric the other is also time-asymmetric.

Clearly as the quality of the space of observables improves, the corresponding space gets smaller but, on the other hand, the quantity of measurement increases and therefore the number of of the measured space states increases as well:

$$O_1 \subset O_2 \Leftrightarrow O_2^{\times} \subset O_1^{\times} \quad \text{or} \quad S_2 \subset S_1$$

From this equation and eq. (3) we obtain[23]:

$$H^{\times} \subset C^{\times} \subset S^{\times} \subset H_{\pm}^{2\times} \tag{5}$$

[23] The Hilbert space has a characteristic property, known as Riesz theorem: it is equal (precisely isomorphic) to its dual:

$$H = H^{\times}$$

So in the worst quality case the space of observables is equal to the space of states. In all the other cases the space of observables is contained in the space of states, since from the last equation and eqs. (3), (5) we have:

$$H_{\pm}^{2} \subset S \subset C \subset H = H^{\times} \subset C^{\times} \subset S^{\times} \subset H_{\pm}^{2\times}$$

In this equation we see, very clearly, how the refinement of the measurement quality increases the state space. Any triplet $H_{\pm}^{\times} \subset H \subset H_{\pm}^{2\times}$, $S \subset H \subset S^{\times}$, $C \subset H \subset C^{\times}$, is known as a Gel'fand triplet [13] or a rigged Hilbert space [11].

2.4. Mixing systems

The arrow of time does not appear in simple systems. They must have some degree of complexity in order that this arrow may appear. In this section we will study the case of classical system, namely non quantum system, where this complexity is called 'chaos'. There are different chaos degrees, and we will be interested in the *mixing chaos*. In fact, this property serves to guarantee the approach of the system to an equilibrium state, which is one of the ways to define the arrow of time. Every physical (mixing) system has a natural tendency to go to an equilibrium final state. Chaos, most likely with mixing properties, is very frequent in mechanical systems. As we will see a (Gibbs) drop of ink spreading in a glass of water, a sugar lump solving in the coffee or an open bottle of perfume diffusing the perfume in the room are all mixing systems. All these motions reach a final homogeneous state of equilibrium. In this final state the percentage of ink, sugar or perfume is homogeneous in the corresponding container (glass, cup or room). This is the definition of mixing evolution: it is an evolution that homogenizes any initial inhomogeneity, in such a way that if this inhomogeneity is the ink drop, this ink will reach a final equilibrium state where it is homogeneously mixed with the water. There is a natural tendency to homogeneous equilibrium through this mixing process, as the examples above show.

A very important and popular mathematical analogue of mixing transformation is the so called 'baker's transformation' that operates in the square space X=1 × 1 (or [0,1] × [0,1]) and it is defined by the following procedure:

I.- Take a square dough of dimensions 1 × 1 (fig. 6).

II.- Squeeze the 1 × 1 square to a 2 × 1/2 rectangle, as the baker does with the dough.

III.- Cut the rectangle vertically into 2 rectangles and

IV.- Pile them up to form another 1 × 1 rectangle.

Then repeat this procedure again and again[24]. The transformation is shown in fig. 6´ (as we will see), where in the first square is the configuration corresponding to the time=0.

Much more complicated mixing evolutions than the baker's transformation can be invented. In fact, the baker's transformation is the simplest of all: it is the simplest model of the famous Gibbs ink drop. Gibbs tried to explain the essence of irreversibility with the ink drop model. If a drop of blue ink is introduced in a glass of water, even if the volume of the ink drop remains constant, we will have, after a while, an homogeneous mixture of bluish water. As we have said, this is the typical final equilibrium state of every mixing evolution. What happens is that the motion of the water is mixing and therefore the ink drop is deformed (even if its volume is constant) in such a way that it is transformed in a set of very thin filaments that are present in every part of the water, giving the sensation that the water has become bluish.

[24] A mathematician would say that in doing so the points of the square will move as:

$$(x,y) \to B(x,y) = \begin{cases} (2x, \frac{1}{2}y) & \text{if } 0 \leq x \leq \frac{1}{2} \\ (2x-1, \frac{1}{2} + \frac{1}{2}y) & \text{if } \frac{1}{2} \leq x \leq 1 \end{cases}$$

at each step.

The growing of this filaments-like structure gives an arrow of time and for Gibbs it is the essence of the direction of the arrow of time.

Now this phenomenon is nicely modelled by the baker's transformation. In fact, let us consider a small rectangle $a \times b$ within the square 1×1 (fig. 6´), let us say a small stain of low quality flour within the dough (that corresponds to the ink drop). The height of the stain will successively become: 1/2*b*, 1/4*b*,... 1/t*b* while the base of the stain will become: 2*a*, 4*a*,...*ta*,... in such a way that the area is conserved. Eventually a time will arrive such that $ta > 1$ and then the stain will be cut in two, and then in four, eight, etc., and at the end it will become a set of horizontal filaments of increasing length and decreasing height (last square of fig. 6´), namely a 'cubist' picture of the ink drop. So the baker's transformation is just a model of the ink drop phenomenon.

If now that we are acquainted with the baker's transformation model, we consider again the much more complicated evolution of the ink drop, we see that the filaments also exist in this motion, even if they are produced with a much more complex geometry (not a cubist one), but with the same essential property: the 'height' of the filaments decreases, and they become thinner, and the 'length' of the filament grows and they become longer. It is clear that the motion of usual water is mixing, according to our definition, as the baker's transformation is. In fact, if the volume of the ink drop is the 1% of the volume of the water, and if the motion is mixing, in the far future every subset will have a 1% of ink and, therefore, the distribution of ink will become homogeneous. As this is the case with the real ink drop, we can conclude that the real motion is mixing.

Going back to fig. 6´, if the shaded $a \times b$ would correspond to the stain of lower quality flour (the analogue of the ink drop), and in this case this flour will fill the 1% of the square $[0,1] \times [0,1]$, it is evident that in the far future any subset of the square, like A, will have a 1% of the low quality flour (even if at t=0 there was not a trace of bad quality flour in A, fig. 6´). Therefore the baker's transformation is also mixing and, as we have said, it is just a model of the ink drop in the glass of water.

But now, for a change, let us consider the evolution towards the past. Let us ask ourselves where the lower quality flour comes from. Going to the past we have the inverse evolution for $a \times b$, and b will become 2*b*, 4*b*,...*tb*,... while a will become 1/2*a*, 1/4*a*,...1/t*a*,..., in such a way that towards the past the height grows and the length decreases. In the far past we will have a set of vertical bands (first square of fig. 6´´). Everything we have said about the evolution towards the future can be repeated with the simple substitution of the horizontal direction by the vertical direction and vice versa. Clearly there is not an essential ontological difference between horizontal and vertical directions. So we will have the behavior just described also towards the past. Also the subset A will have the 1% of lower quality flour towards the past (fig. 6´´). Then the theoretical baker's transformation does not break the past-future symmetry, since we see that the past evolution (before t=0) is similar to the future evolution (after t=0). But the real ink drop does it, since nobody has ever seen an homogeneous mixture of ink and water where the ink concentrates spontaneously in such a way to produce an ink drop (which corresponds to the past evolution of the baker's transformation before t=0). For the ink drop this part of the evolution is physically impossible. So the baker's transformation is a good mathematical model of the ink drop towards the future of t=0, but not towards the past. Let us keep this idea in our minds.

Now, in order to make contact with the measurement process studied in the last two sections, let us consider the fate when $t \to \infty$ of any subset, namely any low quality flour stain in baker's transformation. The horizontal strata will become a set of infinite horizontal straight lines towards the future (fig. 6´´) (known as a horizontal *Dirac's comb*, since the density of the bad quality flour is concentrated in these lines and zero elsewhere, like in the Dirac's δ distribution). Towards the past the set of vertical bands will become a set of infinite vertical lines (known as a vertical Dirac's comb for the same reason) in the limit $t \to -\infty$ (fig. 6´´´). These sets of infinite lines are a superposition of infinite Dirac's distributions δ and they are not the usual low quality flour stains (or ink drops) but idealized generalizations of these stains or drops in the limit of infinite time. Of course, nobody has ever seen these infinite lines. What we really see is the glass of water becoming uniformly bluish or the flour acquiring an uniform quality. From this point on the two schools follow different paths that we will now explain.

2.4.1. The gnoseological school

This school postulates that, as we cannot see with infinite precision, we have a coarse-grained image of the universe, coarse-grained as a photography where the grains of the paper have different colors and, even if they have finite size, they create the illusion of a continuous image because, being the grains so small, we cannot see each one of them individually. Therefore, according to this school, any perception of the measurement must be done considering a set of grains g, of small size ε, such that we cannot measure a smaller length. Then we must average the probability $p(x)$ over each grain and content ourselves to use this averaged probability $\not p\,(x)$. If we want to give a mathematical definition of this idea, we must use functions from the space C. Precisely those which have the following property:

they are zero everywhere but their value is one in just one subinterval of [0,1] of size $\varepsilon < 1$, that we will call the grain g

Such curve will be called the *characteristic curve* (fig. 7) of the grain g and symbolized by $\chi(g)$ (see the mathematical explanation in the footnote[25]). So coarse-

[25] If we make the average (2) of a probability *p(x)* using one of these curves we obtain:

$$<0>_\rho = \sum_g p(x)m(x)\Delta x = \sum_g p(x)\Delta x = <p(x)>_g$$

where the g under the first sum means that we add only in the grain g (i. e. the subinterval of size ε); in the second sum *m(x)* disappears because it is equal to 1 in the grain g, and the symbol in the r. h. s. is the average of *p(x)* in the grain g. Now if we divide the interval [0,1] in grains g_i of size ε, we can define a coarse-graining probability as

$$\not p\,(x) = \chi(g_i) < p(x) >_{gi}$$

graining school is based on functions of quality C. Precisely a set of curves like those of fig 7´.

Let us now go back to the baker's transformation and let us consider a characteristic surface, namely a surface obtained by the multiplication of two characteristic curves in both axis x and y. So in the square $[0,1] \times [0,1]$ the function defined by this characteristic surface is the one of a grain, namely equal to one in the small square $\varepsilon \times \varepsilon$, and zero elsewhere (fig. 7´´). Now we can generalize the average (2) to the two dimensional case: we can consider a probability $p(x,y)$ and define the coarse-graining probability $\bar{p}\,(x,y)$, namely a function where we have substituted the average of the probability $p(x,y)$ in each grain. In the case of the baker's transformation this procedure will give the average of ink or bad flour in each grain. Going back to the beginning of this section, it is evident that the small squares $\varepsilon \times \varepsilon$ are equivalent to the 'grains' of a photography, so ε is the minimal precision that we can use, measure or see.

Then using the curves C we have created observers that measure the average probability density $\bar{p}\,(x, y)$, and a observers space of coarse grain quality Cx for the x axis and C_y for the y axis. Then the observer space can be called $C = C_x \otimes C_y$. For simplicity we have made the length ε equal in both axis, since physically ε is the smallest precision we can measure or see. Then, when time grows, and it goes up to $t \rightarrow +\infty$, and the horizontal strata become smaller than ε, it is quite obvious that the average probability density $\bar{p}\,(x, y)$ becomes a constant and, if the mean value refers to the color of the water or to the mean quality of the flour, we will obtain a constant (precisely 1% for the above examples), which means that we will see homogeneous bluish water or bread dough. Thus the gnoseological school really explains the physical phenomenon. It is our incapacity to measure with an infinite precision the fact that produces the final homogeneous equilibrium state and therefore the arrow of time. On the other hand, following this line the gnoseological school explains much more involved and complex phenomena, like the growing of entropy, as we have said. This entropy grows up to a maximum value when equilibrium is obtained as it should be. In fact, in many respects the gnoseological school is completely satisfactory [15].

The problem is that if we go towards the past, up to $t \rightarrow -\infty$, the same thing happens. When the vertical bands become smaller than ε, we have an homogeneous equilibrium state (with also a maximum of entropy). Then considering the whole process from $-\infty$ to $+\infty$, we go from equilibrium to a state out of equilibrium at t=0 and towards equilibrium again at $+\infty$. Nobody saw this process as a whole, which is equivalent to the concentration of the ink, in a glass where ink and water are homogeneously mixed, to form an ink drop at time t=0 and then to be diffused in the water again. But everybody has seen the second part of it. So coarse-graining applied to a time-symmetric evolution that makes no difference between past and future does not break the time-symmetry and it cannot be the whole story. It only explains the arrow of time from t=0 to $t \rightarrow \infty$, namely a partial arrow of time. It does not explain the global arrow of time, from $t \rightarrow -\infty$ to $t \rightarrow \infty$, since both sides of the evolution are symmetric with respect to t=0. In other words, as the space C is time symmetric it cannot break a time symmetric evolution.

2.4.2. The ontological school

Usually the quality of measurement used in almost all physical theories is Hilbert quality. Then the observers space in the case of the baker's transformation would be $H = H_x \otimes H_y$. With this quality we cannot see the Dirac's distributions, and therefore we cannot see the Dirac's combs. But the ontological school would like to see, e. g. the horizontal Dirac's comb, namely those that really appear when $t \to \infty$ (and do not appear when $t \to -\infty$). Then it must use another observers space: $S_- = H_x \otimes S_y$ because the Schwarz quality in the vertical direction allows us to see this horizontal comb measured by S_y (the vertical comb of the far past cannot be seen, since it is measured by H_x, a quality that does not see the combs). The corresponding space will be S^x, that actually contains the horizontal combs. Then if really the space of physical states has this property, it includes an ontological characteristic, that fixes the equilibrium towards the future, where there is the horizontal comb, but not towards the past since the vertical comb is not contained in the space S_-^x.

Of course we can choose as observers space $S_+ = S_x \otimes H_y$ and as states space S_+^x and we will have the reverse properties: we will have equilibrium towards the past and see the vertical Dirac's comb that appears when $t \to -\infty$, since the time inversion of the horizontal comb gives the vertical comb. This is not the ontological property of the real universe, but the above one, if we postulate that the physical states belong to space S_-^x and not to space S_+^x (we will say more in section 6). The time-symmetry of the baker's transformation is broken by the ontological school using time asymmetric observers contained in the space S_- or, which is the same thing, time-asymmetric states contained in the space S_-^x.

This structure is developed with success in paper [16]. As we now have equilibrium towards the future, as it is actually the case, and we can consider this equilibrium in all details, since it is contained in space S_-^x, we could conclude that the ontological school is superior to the gnoseological one. Somehow it is so, but our analysis shows that it is not purely ontological, since the observers, namely the observation apparatuses, have also an important role in this school, which therefore is not free from a gnoseological component[26].

2.5. The Hardy quality

The baker's transformation is just a didactic example. Therefore the spaces S_-

[26] In baker's transformation the time inversion T is equivalent to change the vertical and the horizontal directions. So the following equations are valid for the observers space:

$S_- \neq S_+, T: S_- \to S_+$

These equations define a *duality* [17]. Thus even if the evolution of the baker's transformation is time-symmetric, it is complex enough to allow the appearance of this dual structure, which turns out to be an essential feature to define time asymmetry in the ontological theories. We have also a duality in the state spaces:

$S_-^x \neq S_+^x, T: S_-^x \to S_+^x$

and S_+ are also didactic examples. Can we find a physical reliable principle to fix the observers space in a unique way? To do so we must find a condition that the observers space must fulfil if we consider their time evolution. So we must consider how the observables evolve with time[27]. Then a logical property to ask to the observers space is that it must be the same when we go towards the future. In fact, the criterion to choose the physical observables cannot change when we go towards the future. Think of a film of an elephant breaking a glass-shop. The camera is the measurement apparatus[28]. We will see that the elephant breaks the glasses, the shelves, and the furniture, going from one state that we can consider physical to another state that we also consider physical, i. e. performing a physical evolution. This will also happen with the ink drop diffused in the water, but in a less spectacular way. As the criterion we use to say that we are seeing the picture in the right direction is the same for all times, therefore we have a first condition:

1.- The space of the observables must be such that any observable should always be contained in this space when it evolves towards the future[29].

In this sense the space O_- is stable towards the future. But this is not a necessary condition towards the past. In fact, if we see the film in the reverse direction, we see non-physical events happening: glasses being reconstructed by the elephant motion (or the ink drop contracting in the glass of water), etc. So, if we go towards the past, the criterion to choose physical observables have changed and we have another condition:

2.- Condition 1 is not necessary towards the past[30].

In this sense the space O_- is unstable towards the past and this is the asymmetry that generates the features we are looking for.

Then essentially from a Beurling theorem [18], [19], [20], [21], we know that conditions 1 and 2 are satisfied if and only if:

$$O_- = qH^2_+ \tag{6}$$

where $q = e^{i\varphi}$ is a phase (a complex number of modulus one) and H^2_+ is the Hardy class function (from above) space[31]. We can disregard the phase q since it can be

[27] There is a time-evolution operator, which we will call e^{iLt}, that transforms the operator $O(0)$, at time $t = 0$, into the operator $O(t)$ at time t. Then the law of the observables evolution towards the future is: $O(t) = e^{iLt}\, O(0)$, where the O are the observables (considered as matrices), $t{>}0$ and L is the so called Liouville operator, while towards the past the last equation reads: $O(-t)=e^{iL\,(-t)}O(0)$.

[28] The measurement apparatuses measure not only position but also velocities. Then the analogy of the film is eloquent but not complete. In fact, we must rather think that each photography of the film also contains information about the velocities (or what we are really considering, as the state at each time is a pair of two successive photographs).

[29] Namely $e^{iLt}O_- \subset O_-$, if $t > 0$, i. e. an admissible observable remains admissible all along the time evolution towards the future.

[30] Namely $e^{iLt}O_- \not\subset O_-$, if $t < 0$, i. e. the property 1 is not valid towards the past.

[31] Really $O_- = H^2_+ = [H^2_+ \;(\mathrm{R},N)]$. For the sake of physicists and mathematicians we add that the variable $\nu \in \mathbb{R}$ is the eigenvalue of the Liouville operator, and N is an auxiliary space that contains all the necessary variables to describe the considered model.

proved that it is irrelevant [22]. So we conclude that our space of physical observables is:

$$O_{-} = H_{+}^{2} \tag{7}$$

Thus we reach the conclusion that the quality motivated in these physical reasons is Hardy quality.

Therefore, even if the usual physical apparatuses belong to the quality *C*, there must be some other physical entities (time asymmetric observables) that perceive the difference between H_{-}^{2} and H_{+}^{2} , e. g. ourselves, since we feel that the past is not the future. Moreover, the time-symmetric apparatuses of quality *C* do not feel this difference since they do not satisfy the property 2. Essentially these measurement apparatuses do not perceive the distinction between past and future but only the direction non-equilibrium → equilibrium, even if, as in the first part of the baker's transformation evolution (from -∞ to 0), the equilibrium is in the past and the non-equilibrium is in the future.

Of course if in all this reasoning we change the roles of past and future we will obtain:

$$O_{+} = H_{-}^{2}$$

so we have the couple O_{-} , O_{+} ,[32] as in the preceding section we had the didactic couple S_{-} , S_{+} . It is clear that, if we deal with a close system (e.g.: the universe), the choice of one of the members of these couples in order to settle the ontological property that defines the arrow of time is conventional. So someone may say that the arbitrary choice of one the members of the couples is made 'by hand'. It is not so. To prove it we will discuss this problem further on the next section.

2.6. The Reichenbach Global System

The arrow of time cannot be a local concept. We have both practical and theoretical reasons that support this statement. i.- From the practical point of view, we have studied very far sections of the universe and we have always used the same physics with the same arrow of time. If the arrow of time would be different in a very far quasar, we would perceive this difference since, e. g., the elementary particles would decay in a different time direction. ii.- From the theoretical point of view we can make the following reasoning: we could conceive two isolated laboratories with different arrows of time, but this fact has never been observed [23]. So, it is conceivable that, given two researchers working in two isolated laboratories, one of them chooses the observables of O_{-} , while the other chooses the observables of O_{+} .

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[32] We have here another duality since:

$O_{-} \neq O_{+}, T: O_{-} \rightarrow O_{+}$ and

$O_{-}^{\times} \neq O_{+}^{\times}, T: O_{-}^{\times} \rightarrow O_{+}^{\times}$

Namely they choose different arrows of time. These two researchers will be very confused when the isolation ceases and they get in contact, since then they will realize that they have different arrows of time. So, to study this problem we must adopt a global view or, what is the same thing, a cosmological model and the simplest of all is the Global System of Reichenbach [24], [25], [23]. In this model every irreversible process (produced in local subsystems or 'branch systems') begins in an unstable state originated, not in a very unlikely fluctuation, but in an unstable state created by the energy coming from other irreversible processes. E. g., the famous Gibbs ink drop in the glass of water was originated in an ink factory, where unstable coal was burned in an oven to extract energy. Coal was originated in geological ages using the energy of the light coming from the sun, where unstable H was burned, and the energy necessary to create H comes from the unstable initial state of the universe, the origin and source of energy of the whole global system. We can represent this global system at the classical level by fig. 8. To introduce our formalism we must go to the statistical level. The easiest thing to do is to go to the quantum level (which is essentially statistical) and to use quantum language. So let us consider a usual scattering process (fig. 9), namely the collision of atoms, nuclei or elementary particles, where some particles coming from an accelerator $a_1, a_2,...$ (fig. 9) hit a target at time t=0, and are transformed and scattered by the collision in outgoing particles $b_1, b_2,...$ Let us cut this process at time t=0 [11] into a creation of unstable states process (fig. 10), similar to the theoretical contraction of the ink drop, where the solutions $a_1, a_2,...$ are incoming ones, and a decaying of unstable states process (fig. 11), similar to the real diffusion of the ink drop, where the solutions $b_1, b_2,...$ are outgoing ones. This last process corresponds to states in the space $O^\times = S$ that naturally decay into an equilibrium state at $t \to \infty$ with a growing entropy. On the contrary the created states correspond to space $O_+^\times = S_+$ (contraction of the ink drop). Actually this space is not realized in the real physical world as such (there is no spontaneous contraction of the ink drop), because before t=0 the system is not just the scattering one, but a more complete one, that includes the acceleration apparatus and the source of energy (like the one of the dotted box 'B' of fig. 12). This is the reason why we have used just the O_- observers, as explained in the previous sections. We cannot use the diagram of fig. 10 because, during the creation process, the system does not exist as such (actually it is a much more complex system, e. g. the ink factory with its oven burning coal). The system really begins to exist at time t=0 (namely the isolated glass of water with the ink drop) and therefore it is only described by the diagram of fig. 11. So really only the second part of the baker's transformation (from 0 to ∞) does exist. That is why you never see the ink concentrating spontaneously in the glass of water. But in the second part of the evolution the past → future direction coincides with the non-equilibrium → equilibrium direction, the arrow of time of the apparatuses of quality C coinciding with the one of the apparatuses of quality H_-^2 , so you can use either one kind or the other. Then you can employ with confidence the usual apparatuses of quality C, which are the real ones since they have a finite accuracy. Also, as the results obtained with the two kinds of quality C and H^2 coincide, there is no physical reason to choose between the gnoseological and the ontological school. There is not a cross-experiment that would prove that one school is right and the other is wrong. Therefore: the difference between the two schools is just philosophical. This is the reason why this paper is addressed to philosophers. Actually the evolution of the universe can be symbolized as a sequence of states in local $O_{-}^{x} = S_{-}$ spaces, as shown in fig. 12, interchanging energy among themselves, and all of them co-ordinated because their energy comes from the unique unstable initial state, namely the cut box in the far left of fig. 12. The whole process can be described using just a global space $O^{Gx} = S^{G}$ (as in the simple cosmological process of ref. [26]). Fig. 12 can be considered the quantum or statistical image of classical Reichenbach global system of fig. 8. As fig. 12 corresponds to O^{Gx}, its specular image corresponds to the time inverted space $O_{+}^{Gx} = S_{+}^{G}$ (fig. 13).

But the physics choosing O^{Gx} is *identical* to the physics choosing O_{+}^{Gx}, because, as there is nothing exterior to the universe, nobody can tell the difference. In fact, all the arrows of time are contained in the object O^{Gx}, so when we change this space by the time-inverted object O_{+}^{Gx}, all the arrows of time change.

Then choosing either O^{Gx} or O_{+}^{Gx}, we would obtain the same time-asymmetric physics with a growing entropy when we go from the initial unstable state in what we will call 'the past', to the equilibrium final state in what we will call 'the future'. A realistic model of the universe is thus obtained. Time-asymmetry is not obtained as an asymmetry of the laws of nature but as an asymmetry of the object under study: precisely the apparatuses measuring the universe which are contained in space O^{Gx}. Then it is a factual and not a legal asymmetry, as announced. This argument also proves that we have not put the arrow of time 'by hand' choosing O^{Gx}, this choice being physically irrelevant, since the same physics is obtained if we choose O_{+}^{Gx}. But the choice must be made, and then an ontological property appears and defines the arrow of time.

2.7. Conclusion

We have concluded that the difference between the two schools is philosophical. So we can ask: Has the arrow of time a gnoseological or an ontological origin? We would say that it is ontological, since the object under study, the space of measurement apparatuses, has the ontological property of being of quality H^2 and it is, therefore, asymmetric (this time asymmetry in the measurement apparatuses will obviously produce the same time asymmetry in the space of states measured by these apparatuses, as explained under eq. (4)). But the arrow is gnoseological too, since we are referring to measurement apparatuses, namely devices to get information. From this point on the research must be continued by the philosophers.

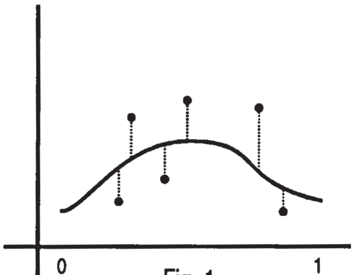


Fig. 1

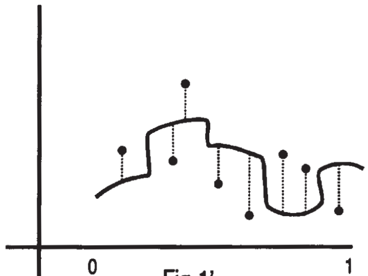


Fig. 1’

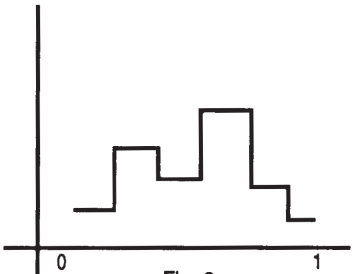


Fig. 2

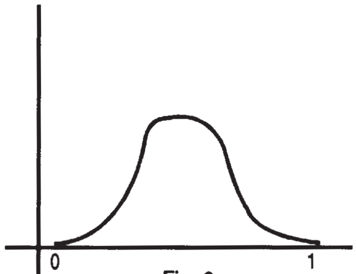


Fig. 3

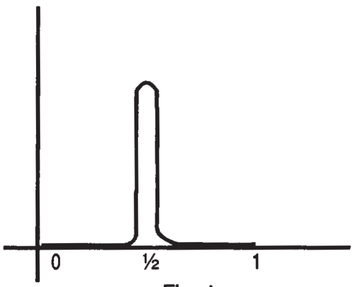


Fig. 4

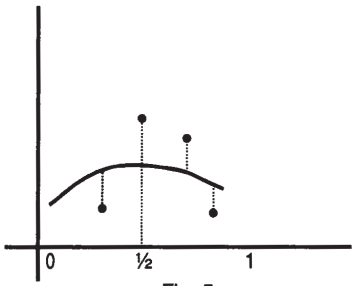


Fig. 5

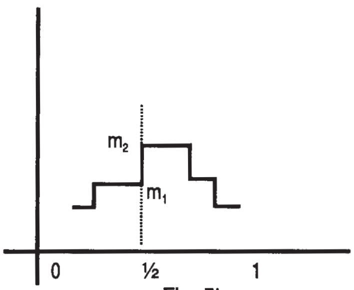
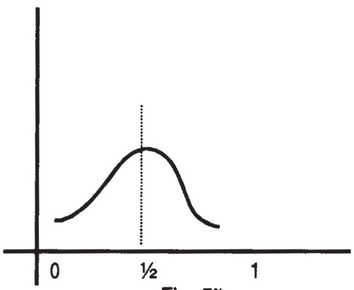


Fig. 5’



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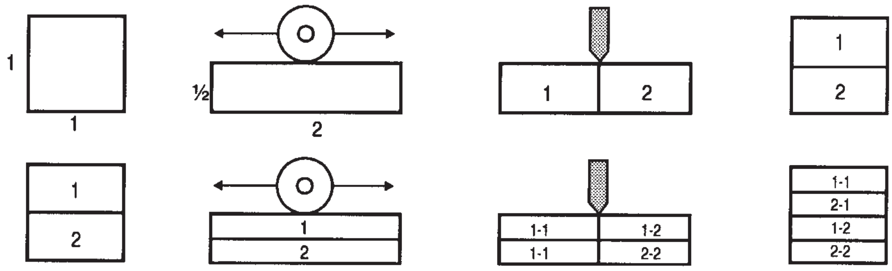

Fig. 6

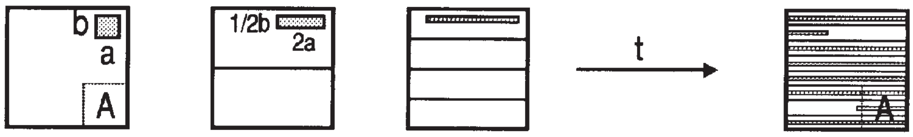

Fig. 6'

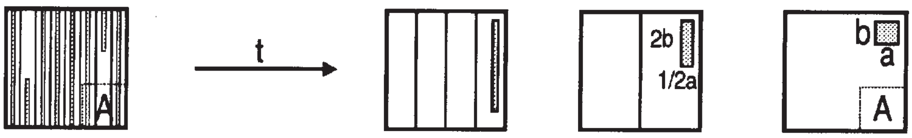

Fig. 6''

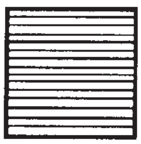

Fig. 6'''

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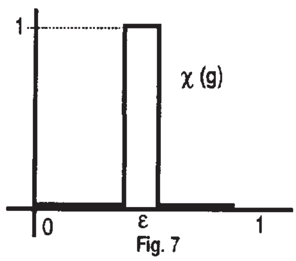


Fig. 7

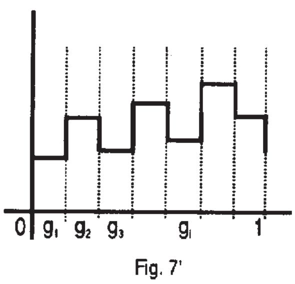


Fig. 7'

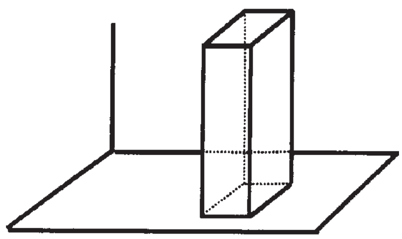


Fig. 7''

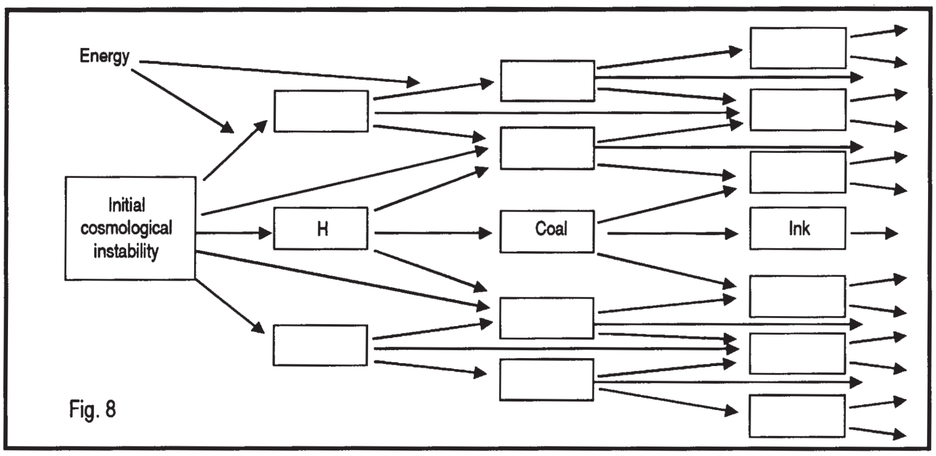


Fig. 8

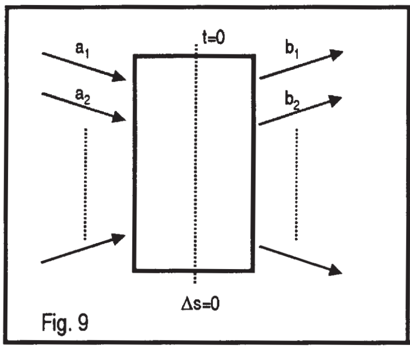


Fig. 9

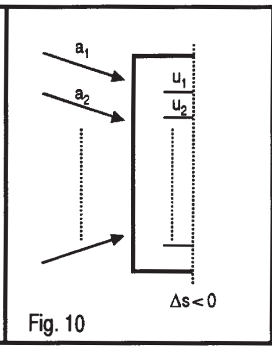


Fig. 10

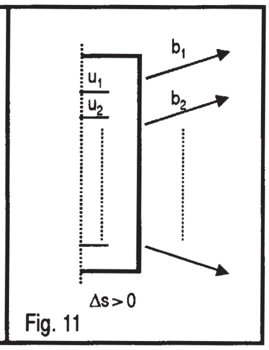


Fig. 11

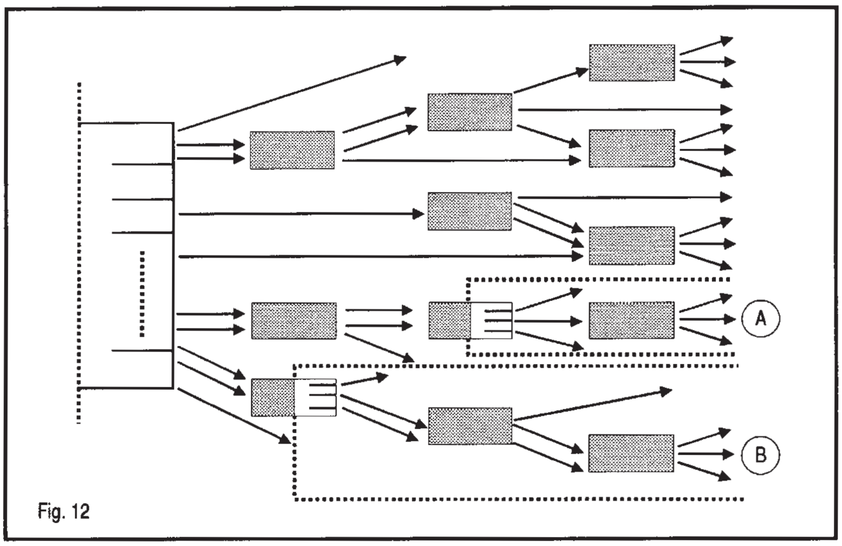
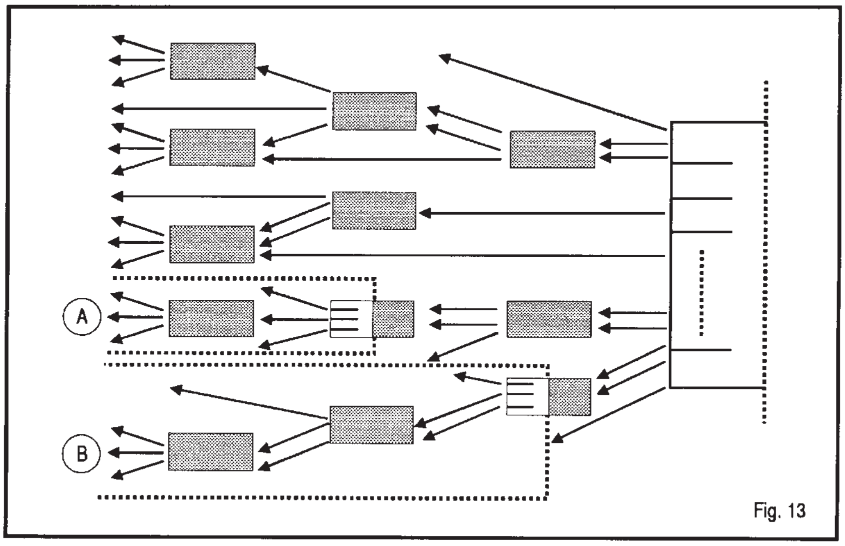


Fig. 12



Figures

Fig. 1. A square integrable function with no jumps. Fig 1´. A square integrable function. Fig. 2. A "coarse-graining" curve. Fig. 3. A Schwarz curve. Fig. 4. A Dirac's distribution. Fig. 5. $m(\frac{1}{})$ measured by a H curve. Fig. 5´. $m(\frac{1}{})$ measured by a C curve. Fig. 5´´. $m(\frac{1}{})$ measured by a S curve. Fig. 6. Baker's transformation. Fig. 6´. The fate of the flour stain towards the future. Fig. 6´´. The fate of the flour stain towards the past. Fig. 6´´´. The fate of the flour at $t \to \infty$. Fig. 6´´´´. The fate of the flour at $t \to -\infty$. Fig. 7. A characteristic curve in the interval [0,1]. Fig. 7´. The coarse-grained curve. Fig. 7´´. A characteristic surface in $[0,1] \times [0,1]$. Fig. 8. Classical Reichenbach diagram. Fig. 9. Scattering process. Fig. 10. Creation process. Fig. 11. Decay process. Fig. 12. Bohm-Reichenbach diagram. Fig. 13. The inversion of fig. 12.

References[33]

[1] A. EDDINGTON, *The Expanding Universe*, Cambridge University Press, Cambridge 1958.

[2] H. D. ZEH, *The Direction of Time*, Springer, Berlin 1992.

[33] Review references are made according to the usual abbreviations. The first number of the review reference, in bold (e. g. **33**) indicates the volume; the second number (e. g. 1330) mentions the first page of the paper.

[3] R.E. SACH, *The Physics of Time Reversal*, Cambridge University Press, Cambridge 1987.

[4] J.L. LEBOWITZ, *Time's Arrow and Boltzmann's Entropy*, in *Physical Origins of Time Asymmetry*, J. J. Halliwell et al. (ed.), Cambridge University Press, Cambridge 1994.

[5] R.V. ZWANZIG, «J. Chem. Phys.», **33**, 1330 (1960).

[6] W. ZUREK, «Physics Today», **44**, 36 (1991).

[7] R. KUBO, M. TODA, O. HASHITSUMO, *Statistical Physics II*, Springer, Berlin 1995.

[8] R.P. FEYNMAN et al., *The Feynman Lectures on Physics*, Addison Wesley, New York 1964.

[9] I. PRIGOGINE, *From Being to Becoming*, Freeman, New York 1980.

[10] L.E. BALLENTINE, *Quantum Mechanics*, Prentice Hall, New Jersey 1990.

[11] A. BOHM, *Quantum Mechanics, Foundations and Applications*, Springer, Berlin 1986.

[12] P. ROMAN, *Advanced Quantum Theory*, Addison Wesley, New York 1965.

[13] I. GEL'FAND, G. SILOV, *Generalized Functions*, Academic Press, New York 1967.

[14] N.N. BOGOLUBOV, A.A. LOGUNOV, I.T. TODOROV, *Introduction to Axiomatic Field Theory*, Benjamin, London 1975.

[15] M. MACKEY, «Rev. Mod. Phys.», **61**, 981 (1989).

[16] I. ANTONIOU, S. TASAKI, «Physica A», **190**, 303 (1991).

[17] B. MISRA, I. PRIGOGINE, M. COURBAGE, «Physica A», **98**, 1 (1979).

[18] M. COTLAR, *Teoremas espectrales, modelos funcionales y dilataciones de operadores en espacios de Hilbert*, Cursos de matemática del IAM, 5, CONICET, Buenos Aires 1991.

[19] M. ROSENBAUM, J. ROVNYCK, *Hardy Classes and Operators Theory*, Oxford University Press, Oxford 1985.

[20] P.B. LAX, R.S. PHILLIPS, *Scattering Theory*, Academic Press, New York 1967.

[21] M. CASTAGNINO, S. IGURI, A. ORDOÑEZ, E. GUNZIG, *Minimal Irreversible Quantum Mechanics, Foundations and Applications to Kolmogorov-Lax-Phillips Systems*, in preparation, 1997.

[22] M. CASTAGNINO, E. GUNZIG, *Minimal Irreversible Quantum Mechanics, An Axiomatic Formalism*, submitted to «Int. Jour. Theo. Phys.», 1997.

[23] M. CASTAGNINO, *The Global Nature of Time Asymmetry and the Bohm-Reichenbach Diagram*, Proceeding G21, Goslar 1996, A. Bohm (ed.), World Scientific, Singapore 1997 (in press).

[24] H. REICHENBACH, *The Direction of Time*, University of California Press, Berkeley and Los Angeles 1956.

[25] P. DAVIES, *Stirring up trouble*, in *Physical Origins of Time Asymmetry*, J. J. Halliwell et al. (ed.), Cambridge University Press, Cambridge 1994.

[26] M. CASTAGNINO, *The Mathematical Structure of Quantum Superspace as a Consequence of Time Asymmetry*, «Phys. Rev. D», **57**, 750, 1998.

* * *

Abstract: *L'articolo affronta il problema della direzione del tempo fisico in una prospettiva interdisciplinare tra la filosofia e la scienza sperimentale. Nella parte scientifica, M. Castagnino imposta la questione dal punto di vista di una teoria della misurazione inerente alla metodologia della fisica, in rapporto specialmente ai sistemi dinamici complessi, nei quali si manifesta l'asimmetria temporale. Dall'analisi di due approcci, gnoseologico e ontologico, si conclude che la freccia del tempo della fisica, nei livelli considerati, contiene una mediazione gnoseologica ma anche un elemento ontologico. Nella parte filosofica, J.J. Sanguineti presenta in modo qualitativo il contenuto della sezione scientifica e sottolinea il ruolo di certe scelte filosofiche nel campo scientifico, tenendo conto però della differenza tra l'impostazione realista e positivista.*